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Nucleon strangeness as the response to a strangeness-sensitive probe in a class of hadron models

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Abstract

On top of its valence quarks, the full nucleon ground state may contain appreciable admixture of $s\bar{s}$ pairs already at small momentum transfers. This paper discusses strangeness in the mean–field type of nucleon models, and exemplifies this by explicit calculations in the MIT bag model enriched by the presence of instantons. We calculate the instanton contribution to the strangeness in the MIT bag (on top of the standard contribution to strangeness found in that model). Although we do it in an essentially perturbative way, we present a detailed derivation of the formula expressing nucleon matrix elements of bilinear strange quark operators, in terms of a model valence nucleon state and interactions producing quark-antiquark fluctuations on top of that valence state. We do it in detail to clarify our argument that in the context of the mean–field type of quark models (where a Fock state expansion exists and where the nucleon state can be constructed out of single-quark states), the resulting formula acquires a significance beyond perturbation theory. The derivation combines the usage of the evolution operator containing a strangeness source, and Feynman-Hellmann theorem.

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1 Introduction

For quite some time already, a number of investigators has been considering possibly nonzero strange matrix elements of non-strange particles, nucleons [1–13]. Such intriguing discussions received additional impetus from the experimental [14] and theoretical investigations of the related problem, namely surprisingly small fraction of the nucleon spin carried by the quarks [12, 13, 15–21], [6]. The intense research on nucleon strangeness continued into nineties up to the present day; references [22–47] are just some of the examples. More precisely, matrix elements in question are $\langle N|\mathcal{O}_s|N\rangle$, where $|N\rangle$ is the nucleon state and $\mathcal{O}_s$ is an operator containing strange (s) quark fields bilinearly. In this section, the integration over the three-space is understood in the matrix element. Later, we will indicate the integration explicitly where needed. We will be concerned with $\mathcal{O}_s = \overline{s}\Gamma s$, where Γ is an arbitrary matrix in the spinor space.

Namely, although the *valence* component $|N_0\rangle$ of the full nucleon state $|N\rangle$ contains only u and d quarks, quark-antiquark fluctuations include the $s\overline{s}$ component, allowing

$$\langle N|\overline{s}\Gamma s|N\rangle \neq 0 \tag{1}$$

even though the net strangeness of the nucleon state $|N\rangle$ is of course zero. Some of these matrix elements may be surprisingly large, possibly pointing to some effects not expected in the naive quark model of hadrons. *E.g.*, the strange scalar density inside the nucleon is connected with the experimentally measured $\pi - N$ sigma-term through the ratio

$$y = \frac{\langle N|\overline{s}s|N\rangle}{\frac{1}{2}\langle N|\bar{u}u + \overline{d}d|N\rangle}. \tag{2}$$

For example, see [1]- [13], [22,28,29,38]. A review [38] containing discussions of a very complete set of original references, estimates $y = 0.22 \pm 0.16$. Also, EMC experiment [15] provides evidence that $\langle N|\bar{s}\gamma_\mu\gamma_5 s|N\rangle$ is possibly relatively large.

In the study of the long-debated issue of nucleon strangeness, the usage of nucleon models is still important. This of course rises the question of the model dependence — even concerning the results on what is the basic mechanism behind the effect. For example, an analysis of Steininger and Weise [45] of the scalar strangeness of the nucleon performed in the framework of the Nambu Jona-Lasinio (NJL) model, obtained a very small upper bound on the scalar strangeness from the NJL model with four-momentum cutoff, a larger but still modest upper bound on it from the NJL model with a three-momentum cutoff, but dramatically higher scalar strangeness arises when instanton-induced interaction among quarks dominates. In addition, these authors found only a small contribution, less than 3%, from kaon loops. On the other hand, kaon loops are the basic mechanism for

generating the nucleon strangeness in some other approaches (see, *e.g.*, [48] and Refs. in Sec. 2.1 in [49], or discussion in Forkel *et al.* [50]). Other examples are provided by the strangeness electric mean-square radius, the sign of which is positive according to [46, 51, 52], but negative in some other approaches [13, 26, 48, 50, 53–56], or the strangeness nucleon magnetic form factor, for which predictions of various models and analyses range from $+0.37$ [51,52] over positive [13,55] to various negative values [13,26,49,53–56] all the way down to possibly -0.75 ± 0.30 [53, 54].

This illustrates the motivation to investigate such issues further, in as large number of different approaches as possible, attempting to decrypt what is the physics behind model dependence. In the present paper, we formulate a framework which will in principle enable comparison of such results [45] with corresponding results in a wider range of complementary models. We also want to propose a framework which will be applicable not only to the scalar strangeness, but more generally. Below, we will give an expression for $\langle N| : \bar{s}\Gamma s : |N\rangle$ where : ... : denotes normal ordering with respect to the non-perturbative vacuum $|0\rangle$:

$$: \bar{q}\Gamma q : = \ \bar{q}\Gamma q - \langle 0|\bar{q}\Gamma q|0\rangle \ . \tag{3}$$

Γ is an arbitrary matrix in the spinor space, say $\Gamma = 1_4, \gamma^\mu, \gamma_5, \gamma^\mu\gamma_5, \sigma^{\mu\nu}, ...$, depending on whether one is interested in the scalar, pseudoscalar, vector, axial, and for some purposes maybe even tensor, pseudotensor, *etc.*, ... strangeness of the "full" (model) nucleon state $|N\rangle$ which may contain $s\bar{s}$ pairs. Any interaction (call it $\mathcal{L}_I$) which can produce $s\bar{s}$ pairs can lead to such a nucleon state containing an intrinsic strangeness component.

That matrix elements $\langle N|\bar{s}\Gamma s|N\rangle$ can be significantly different from zero, is not very surprising in nonperturbative QCD in the light of its non-vanishing quark scalar condensates – the *finite* vacuum expectation value[1] of $\bar{s}s$ is actually approximately as large as for the non-strange quarks: $\langle 0|\bar{s}s|0\rangle \approx \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle \approx (-240\text{MeV})^3$. The MIT bag model provides a good illustration how this leads to a large $\langle N|\bar{s}s|N\rangle$ [1]. However, there may also be $s\bar{s}$-pairs other than those from the QCD vacuum condensate, so that normal-ordered strange operators can in principle also have non-vanishing nucleon matrix elements. Fig. 1 illustrates how a non-vanishing value of not only $\langle N|\bar{s}\Gamma s|N\rangle$, but also $\langle N| : \bar{s}\Gamma s : |N\rangle$, can then get a contribution from these $s\bar{s}$-pairs not from the vacuum condensate: at the instant $t = t_0$ the composite nucleon is hit by an external probe (*e.g.*, a neutrino [19]) with the coupling Γ to the strange quarks. Due to an interaction capable of producing $s\bar{s}$-fluctuations, the nucleon state $|N\rangle$ at the time-slice

[1] This is what is often – *e.g.*, in O.P.E. – denoted by $\langle 0| : \bar{q}q : |0\rangle$ $(q = u, d, s)$, but where the normal ordering is with respect to the *perturbative* vacuum, so that it does not vanish in the nonperturbative vacuum. We reserve the notation : ... : for the normal ordering with respect to the non-perturbative QCD vacuum.

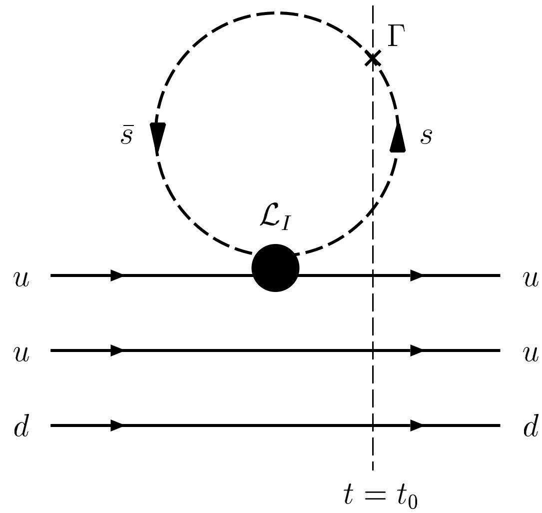

Figure 1: Non-vanishing nucleon strangeness due to a response of the valence nucleon state to a strangeness source at Γ (denoted by $\times$), *i.e.* to a probe coupled to strange quarks through Γ. More precisely, this graph is that part of the nucleon response which arises only through one interaction $\mathcal{L}_I$.

$t = t_0$ obviously contains not only the valence quarks uud, but also the s–quark loop to which the external probe can also couple. Let us *schematically* write down the full nucleon (proton) state which is also coupled to the strangeness-sensitive probe:

$$|N\rangle = \frac{1}{\mathcal{N}}\left(\sum_{X=0}^{\infty} C_X |uud\,X\rangle + \sum_{X=0}^{\infty} C_{s\bar{s}X} |uud\,s\bar{s}\,X\rangle\right) \equiv \frac{1}{\mathcal{N}}\Big(|N_0\rangle + |\delta N\rangle\Big)\,, \tag{4}$$

where X (starting from $X = |0\rangle$ standing for the complicated non-perturbative QCD vacuum) symbolizes any number of various perturbative *and* non-perturbative gluon configurations but also any number of quark-antiquark pairs, including strange pairs which escaped detection by this probe. These complicated configurations "dress" quarks ($q = u, d, s...$) into their effective counterparts – constituent quarks $\mathcal{Q} = \mathcal{U}, \mathcal{D}, \mathcal{S} \ldots$. (In terms of the constituent quarks, this part unperturbed by the strangeness-sensitive probe, is just the valence part: $|N_0\rangle = |\mathcal{UUD}\rangle$. That $\langle N_0| : \bar{s}\Gamma s : |N_0\rangle = 0$ is especially obvious in terms of the constituent quarks.) The *one* strange pair detected at Γ, has been explicitly denoted by $s\bar{s}$ in the $|\delta N\rangle$-part of the nucleon state perturbed by the probe. $|\delta N\rangle$ can be viewed as the response of $|N_0\rangle$ to the weakly coupled strangeness-sensitive probe. (The coefficients $C_X, C_{s\bar{s}X}$ denote the amplitudes of states with various admixtures X or $s\bar{s}\,X$. $\mathcal{N}$ is the normalization.) This response makes possible that in principle the total nucleon Γ–strangeness $\langle N|\bar{s}\Gamma s|N\rangle$ also receives a nonvanishing contribution

from the non-vacuum channel $\langle N| : \bar{s}\Gamma s : |N\rangle$.

However, the question is how to get the nucleon state in specific enough terms in order to have a calculable expression for $\langle N| : \bar{s}\Gamma s : |N\rangle$. To get it *exactly* would probably be tantamount to solving nonperturbative QCD — consider, for example, that the Fock state expansion itself must be built upon the nonperturbative QCD vacuum ($X = 0$), which is unknown. This is why we said that (4) is only a schematic, illustrative expression. Therefore, one obviously has to rely on models to a large extent. One seemingly more viable approach could for example be to *model* $|N_0\rangle$ in a conventional way in terms of *only* non-strange effective quarks (so that $\langle N_0| : \bar{s}\Gamma s : |N_0\rangle = 0$ even though $\langle N_0|\bar{s}\Gamma s|N_0\rangle \neq 0$ at least for $\Gamma = 1$ due to the strange vacuum condensate), and then use appropriate interactions $\mathcal{L}_I$ to infect it by $s\bar{s}$-fluctuations and thereby produce $|\delta N\rangle$ - say, using perturbation theory if $\mathcal{L}_I$ happens to be perturbative. This is what (6) in the next section amounts to. However, in that section we also point out why one cannot proceed quite so straightforwardly, and then give our alternative formulation with the formula for matrix elements of strange operators. How this expression can be evaluated is explained in more detail in the third section, where we also explain why we are motivated to investigate the case of the instanton-induced interaction. The evaluation of various strange densities — with this $\mathcal{L}_I$, and in a concrete nucleon model — is carried out in the fourth section. We conclude in the fifth section.

2 Formulation of a model approach to nucleon strangeness

As pointed out by Forkel *et al.* [50], the ("naive") absence of virtual $q\bar{q}$ pairs in the hadron wave functions in the models based on constituent quark core, makes the treatment of nucleon strangeness in such models far from straightforward. Since the approach presented below is complementary to other ones which have also used dressed quarks in some way (*e.g.*, [17,45,50]), we first give a review of some quark-model notions that will be relevant below.

The purpose of working with hadron models is, of course, not to solve but to *imitate* the horrendously complicated non-perturbative low-energy QCD. Accordingly, various gluon field configurations (*e.g.*, instantons, or those configurations responsible for confinement) and polarization clouds of fluctuating $q\bar{q}$ pairs (all symbolized by X's in (4)), as well as all interactions between all these fundamental constituents, are taken into account through parameters of some nucleon model and appropriate wave functions for dressed, *effective* quarks and antiquarks $\mathcal{Q} = \mathcal{U}, \mathcal{D}, \mathcal{S}$. Examples may be various constituent quark models, where baryons consist just of valence quarks which are however *constituent* quarks, quasi-particles which come about through

dressing of the current quarks by QCD — *i.e.*, in other words, by our X's. Or, it may be the MIT bag model, where these long-range nonperturbative QCD effects lead to, or are partially parametrized by, a confining cavity containing again a fixed number of effective valence quarks (and antiquarks, in the case of mesons and "exotic" $qqqq\bar{q}$ baryons). Choosing a definite model of the hadron structure implies also the choice of the model wave function basis $q_K(x)$ in which to expand the quark fields $q(x)$ $(q = u, d, s)$ in terms of creation $(\mathcal{U}_K^\dagger, \mathcal{D}_K^\dagger, \mathcal{S}_K^\dagger)$ and annihilation $(\mathcal{U}_K, \mathcal{D}_K, \mathcal{S}_K)$ operators of dressed quarks and antiquarks. (K stands for the set of quantum numbers labeling a model quark state. For the expansion specific to the MIT bag-model see the Appendix.)

It is then clear, for example, that the nucleon $|\mathcal{UUD}\rangle$ (when all three of these effective quarks are in their ground states), is nothing but our $|N_0\rangle$ from (4) except that all the mess of fluctuations X is by some model parametrization lumped into dressing of valence quarks $\mathcal{UUD}$, as well as into effective model interactions, or a mean field they feel. Obviously, the idea here is to represent hadrons as composed of a fixed, well-defined number of dressed valence quarks (and antiquarks), bound by effective model interactions which sum up reasonably successfully the fundamental QCD ones. The most simplified, but illustrative case is when these model quasiparticles are moving in an average, mean field Φ. To be sure, these model interactions (and/or mean field), as well as the effective, dressed quarks $\mathcal{Q}$, are assumed to be "produced" by all relevant interactions between quarks at more fundamental levels *including* presently interesting strangeness-producing interactions $\mathcal{L}_I$. "Produced" here of course means that we *modeled* them, not "calculated" from these underlying relevant interactions. So, they (including $\mathcal{L}_I$) are assumed to be accounted for through modeling.

Note, however, that this approach does not say what would be the model representation (or parametrization) of $|\delta N\rangle$, as it does for $|N_0\rangle = |\mathcal{UUD}\rangle$. Of course, in the spirit of all said above, we can write $|\delta N\rangle \sim |\mathcal{UUDS\bar{S}}\rangle$, and writing this is even quite useful for reminding us that *i)* the fluctuating strange (anti)quarks — being embedded in the nucleon — also have to be dressed in the way prescribed by whatever model is applied, including being in one of the model single-quark eigenstates, and that modeling effectively takes care of all their interactions (except of course the interactions induced by their coupling at Γ to their source, a probe sensitive to strangeness), and *ii)*, that all other fluctuations (X's in (4)) are lumped in the dressing, so that the only allowed quark-antiquark fluctuation is $\mathcal{S\bar{S}}$, which has its source in the external strangeness-sensitive probe at Γ. However, in contradistinction to, *e.g.*, $|N_0\rangle = |\mathcal{UUD}\rangle$ which is unambiguous because we know that there all quarks are in their model ground states (and corresponding quantum numbers on $\mathcal{UUD}$ are suppressed for brevity of the notation, but known in principle), $|\delta N\rangle \sim |\mathcal{UUDS\bar{S}}\rangle$ is just a generic formula, a useful mnemonic as

just described, because in this case we *do not* know in what states these five constituents are. In principle, $|\delta N\rangle$ is a superposition of all possible such states, encompassing exotic baryons with $\mathcal{UUDS\bar{S}}$ contents and ordinary strange baryons coexisting with kaons, as well as nucleons with $s\bar{s}$-mesons — most notably ϕ-mesons.

So, let us call H_0 the Hamiltonian responsible for the formation of hadron states composed of definite, fixed numbers of quarks — and possibly antiquarks. In the simplest case, we can imagine H_0 as consisting of a sum of one-body quark operators, say typically of the effective quark kinetic energy operator K and the mean, or self-consistent, field Φ in which the dressed valence quarks would move. In any case, H_0 defines the nucleon model — possibly together with some other ingredients (like the confining boundary condition in bag models, for example). The valence nucleon state $|N_0\rangle$ would then be the ground eigenstate, and $|k\rangle$ would stand for all possible higher eigenstates of H_0,

$$H_0|N_0\rangle = E_{N_0}|N_0\rangle, \qquad H_0|k\rangle = E_k|k\rangle, \qquad E_k > E_{N_0}. \tag{5}$$

For example, H_0 could be the static bag model Hamiltonian. $|N_0\rangle$ would then be the bag model nucleon in its ground state, and $|k\rangle$ all higher bag states with a definite number of constituents, including also "bagged" $\mathcal{UUDQ\bar{Q}}$ exotic baryons and the product meson-baryon bag states such as $|k\rangle = |\mathcal{UDQ}\rangle|\mathcal{U\bar{Q}}\rangle$.

What H_0 cannot do is to produce $s\bar{s}$ fluctuating pairs. For that we have to invoke $\mathcal{L}_I$, or its corresponding Hamiltonian H_I, as by assumption they can produce $s\bar{s}$ excitations on top of $|N_0\rangle$. To clarify that introducing $\mathcal{L}_I$ does not lead to double-counting, let us repeat that H_0 is just a model Hamiltonian, the parameters of which should mimic the effects of full, true non-perturbative QCD as much as possible. For example, if H_0 is the Hamiltonian of the non-relativistic naive constituent quark model, it must contain the postulated mass parameter of the constituent quark mass $M_{\mathcal{Q}} \approx M_{N_0}/3$. The corresponding quantity in the true theory, the dynamically generated quark mass, is (in principle) the result of all possible QCD interactions, so that the interactions related to H_I can, in real QCD, also contribute to this mass by contributing to the $s\bar{s}$-fluctuations. The dynamically generated non-strange quark mass must be close to the model constituent quark mass parameter $M_{\mathcal{Q}}$ sitting in H_0, and only in such implicit, indirect ways are interactions like H_I "present" in H_0. However, they are not present explicitly, and, in fact, H_0 cannot produce any $s\bar{s}$ fluctuations at all. Therefore, if we want to study the $s\bar{s}$ fluctuations, we must introduce H_I to enrich the model nucleon with $\mathcal{S\bar{S}}$-fluctuations on top of $|N_0\rangle$. Correspondingly, $\mathcal{L}_I$ (and thus also H_I) contains strange quark field operators bilinearly so that it can connect $|N_0\rangle$ and $|\delta N\rangle$ containing $s\bar{s}$ pairs. (This also implies $\langle N_0| : H_I : |N_0\rangle \equiv \Delta^{(1)} E_N = 0$ regardless of what precisely this interaction

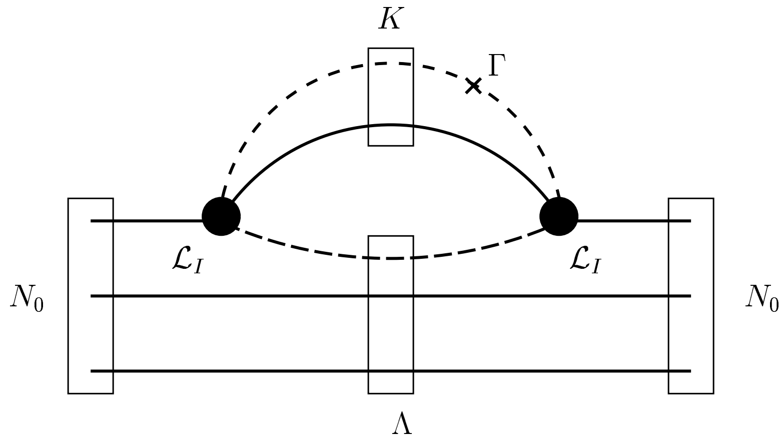

Figure 2: A response of the valence nucleon state $|N_0\rangle$ to a strangeness source at Γ through two interactions $\mathcal{L}_I$. This type of contribution can be associated with the kaon-loop contribution to the nucleon strangeness (a possible $K\Lambda$ intermediate state is therefore indicated).

is. This will be important in (10) below, for the first-order shift $\Delta^{(1)}E_N$ and the third-order shift $\Delta^{(3)}E_N$.) In our figures, this interaction is depicted as a two-body operator, where a strange quark bilinear is combined with a non-strange bilinear. This may be, for example, the two-body part of $\mathcal{L}_{inst}$, the instanton-induced interaction[2]. On the other hand, Steininger and Weise [45] studied the three-body part [59, 60] of instanton-induced interaction (which part they call $\mathcal{L}_6$). Nevertheless, the arguments here are completely general and encompass such cases too; one would just have to do some obvious modifications in our figures. (For example, in Figs. 1 and 2, such a $\mathcal{L}_I$ would, in addition to the strange quark loop, straddle not just one but two valence quark lines of different flavours.) So, $\langle N| : \bar{s}\Gamma s : |N\rangle$ could be evaluated if $|\delta N\rangle$ could be found. But how? For instance, it is easy to see that straightforward application of perturbation theory to find $|\delta N\rangle$, where

$$|\delta N\rangle = \sum_{k \neq N_0} \frac{\langle k|H_I|N_0\rangle}{E_{N_0} - E_k} |k\rangle + \ldots, \tag{6}$$

is hardly viable even in those cases when H_I would be truly perturbative. Namely, it necessitates the summation over intermediate states $|k\rangle$ (some of which must contain $s\bar{s}$-pairs, in order to give $\langle N| : \bar{s}\Gamma s : |N\rangle \neq 0$), which is very hard to handle in practice. Admittedly, Geiger and Isgur [46] have recently succeeded in performing such a straightforward perturbation calculation of the proton strangeness (i.e., using (6)). However, in order to

[2]See, *e.g.*, $\mathcal{L}_{inst}$ of Shifman, Vainshtein and Zakharov [57], or its version used in [58].

make their calculation tractable, they were forced to model hadrons as simple harmonic oscillators. Also, the choice where to put a cutoff, i.e. which intermediate hadron states $|k\rangle$ to discard, is more ambiguous than when working with quarks.

Fortunately, the alternative formulation through the evolution operator is also possible. We formulated and used it in Ref. [47]. Here we present a more detailed derivation of the expressions used there. More concretely, we will combine the Feynman-Hellmann theorem [61, 62] with the usage of the evolution operator containing the Hamiltonian with the source of the strange current of interest. This is because in cases like this one, where we would like to avoid the need to construct $|\delta N\rangle$ explicitly, the method of sources is especially helpful. (See, *e.g.*, [63], pages 89,90.) Of course, in this approach $|\delta N\rangle$ is the response of $|N_0\rangle$ to the external probe which is the source of a strange current, and, naturally, we had this approach in mind already when we wrote schematically the full nucleon state coupled to a strangeness source as (4). This way we will not need $|\delta N\rangle$ explicitly. Instead, we will obtain the nucleon matrix element of this current as the response (to the current source) of the *transition amplitude* of the model ground state $|N_0\rangle$ at $t \to -\infty$ into itself, but at $t \to +\infty$.

We will use normal-ordered operators in order to get an expression for $\langle N|:\bar{s}\Gamma s:|N\rangle$ which can be non-vanishing due to the strange densities that may exist in the nucleon *on top* of the vacuum condensate densities that exist in the QCD vacuum. We are then in principle able to evaluate this matrix element because we assume we can represent the ground state $|N_0\rangle$ by a known nucleon model. Whatever we do below could be done also without normal ordering, but then the analogous expression would include the strangeness due to the strange condensate in the complicated QCD vacuum, and since we neither know the QCD vacuum state nor presently address its modeling, we cannot evaluate this expression. How to find the vacuum part of nucleon strangeness, is an issue that depends on the relation of each specific hadron model with the QCD vacuum quark condensates. For example, this vacuum contribution to the scalar strangeness was found in the MIT bag model by Donoghue and Nappi [1], while the expression for $\langle N|:\bar{s}\Gamma s:|N\rangle$ evaluated in our Sec. 4, is the strangeness induced in the MIT bag nucleon valence ground state in addition to the vacuum contribution.

Using the method of sources in combination with the model approach will also enable us to use the perturbative expansion of the evolution operator only formally; since there are plausible physical arguments, which are *different* from the usual argument (used in Ref. [47]) of "smallness of the perturbation", that higher orders should be neglected, the resulting formula for strange nucleon matrix elements should be applicable even when the interactions $\mathcal{L}_I$, which lead to their non-vanishing values, is not really perturbative. These arguments are a novel element with respect to Ref. [47]. Namely, as already emphasized, the state used here as the ground state, $|N_0\rangle$, is fully

determined by some model Hamiltonian H_0 which also sums up the effects of $\mathcal{L}_I$ in dressing of the constituent valence quarks, so that this interaction with strange quarks is not explicitly present — just as the strange quarks are not explicitly present in $|N_0\rangle$. However, $\mathcal{L}_I$ is again *induced* by the external strangeness source, because it brings in a $s\bar{s}$ pair, which is not hidden in the modeled dressing polarization clouds (X's), and which must be absorbed via $\mathcal{L}_I$ by the valence quarks so that the system is returned to $|N_0\rangle$. However, it turns out that a recognizable response within a defined nucleon model can be obtained only for a *limited* number of $\mathcal{L}_I$-vertices, otherwise a double-counting occurs through dressing of already dressed quarks. All this will be explained more concretely below, after the derivation, which we present in more detail than in Ref. [47] so that the common points as well as differences with respect to Geiger and Isgur [46] are clear.

So, let us define another, auxiliary perturbation Hamiltonian H' by adding to H_I a source term for the strange operator we want to calculate in the "full" nucleon state:

$$H' \equiv H_I + \lambda \otimes \langle \overline{s}\Gamma s\rangle, \tag{7}$$

where $\langle \overline{s}\Gamma s\rangle$ is the convenient abbreviation

$$\langle \overline{s}\Gamma s\rangle \equiv \int \overline{s}(x)\Gamma s(x)\, d^3x\,. \tag{8}$$

(However, in all matrix elements $\langle N|:\overline{s}\Gamma s:|N\rangle$ above and below, the three-space integration over $\overline{s}(x)\Gamma s(x)$ is understood!) The generic form $\lambda\otimes\Gamma$ can mean any of the cases $\lambda 1_4, \lambda_\mu\gamma^\mu, \lambda_{5\mu}\gamma^\mu\gamma_5, \lambda_{\mu\nu}\sigma^{\mu\nu}, \dots$. The Hamiltonians are normal ordered. It is usually implicitly understood, but, for clarity, we will indicate normal ordering explicitly everywhere in the remainder.

Obviously, the Feynman-Hellmann theorem applied to our $H(\lambda) = H_0 + H'(\lambda)$ says that the sought strange matrix element of the full nucleon state is

$$\langle N|\int :\overline{s}(x)\Gamma s(x):\, d^3x|N\rangle = \langle N|\frac{\partial :H(\lambda):}{\partial\lambda}|N\rangle\Big|_{\lambda=0} = \frac{\partial E_N(\lambda)}{\partial\lambda}\Big|_{\lambda=0}, \tag{9}$$

where we also took the physical limit of the vanishing strength ($\lambda = 0$) of the external strangeness source.

Since $\Delta^{(1)}E_N = 0$, the perturbed ground-state energy $E_N(\lambda)$ is given by

$$\begin{aligned} E_N(\lambda) = E_{N_0} &+ \sum_{k\neq N_0} \frac{\langle N_0|:H'(\lambda):|k\rangle\langle k|:H'(\lambda):|N_0\rangle}{E_{N_0}-E_k} \\ &+ \sum_{k,l\neq N_0} \frac{\langle N_0|:H'(\lambda):|k\rangle\langle k|:H'(\lambda):|l\rangle\langle l|:H'(\lambda):|N_0\rangle}{(E_{N_0}-E_k)(E_{N_0}-E_l)} + O[H'(\lambda)^4] \\ &\equiv E_{N_0} + \Delta^{(2)}E_N(\lambda) + \Delta^{(3)}E_N(\lambda) + O[H'(\lambda)^4]. \end{aligned} \tag{10}$$

$O[H'(\lambda)^4]$ stands for the fourth and higher orders, which, as we will argue soon, turn out not to contribute to (9). The presence of the sums over hadronic intermediate states $|k\rangle$ of (5), similarly as in the approach of Geiger and Isgur [46] should be noted. One can then easily understand how we can capture similar aspects of the physics of nucleon strangeness in our respective approach. Now, how do we expect to render the strangeness in (9) calculable, when (10) contains sums over intermediate states, and, as pointed out above, handling them is precisely the difficulty that makes the conventional perturbative approach (as in (6)) useless in practice? What helps here is that we can relate $\frac{\partial E_N(\lambda)}{\partial \lambda}$ to the nucleon matrix elements of the evolution operator $U(t_2, t_1)$, whose perturbation expansion is

$$U(t_2,t_1) = 1 + \sum_{n=1}^{\infty} U^{(n)}(t_2,t_1) = \hat{T}\left\{1 + \sum_{n=1}^{\infty} \frac{i^n}{n!}\Big[\int_{t_1}^{t_2} :L_{\rm int}(t): dt\Big]^n\right\}. \quad (11)$$

$\hat{T}$ denotes the time ordering operator and $L_{\rm int}(t) = \int \mathcal{L}_{\rm int}(\mathbf{x},t) d^3x = -H_{\rm int}(t)$ is the interaction Lagrangian. In our case, we should replace the interaction in the integrand with the form containing the strangeness sources, like in the definition of H', (7):

$$L(t)_{\rm int} \to L'(t) = L_I(t) - \lambda \otimes \langle \overline{s} \Gamma s(t) \rangle = \int d^3x \Big[\mathcal{L}_I(x) - \lambda \otimes \overline{s}(x) \Gamma s(x)\Big]. \quad (12)$$

For definiteness, let us from now on specialize $\lambda \otimes \Gamma$ to $\lambda_\mu \gamma^\mu$, *i.e.*, suppose that we are after the vector strangeness of the nucleon. It is trivial to reformulate what follows for any other possible $\lambda \otimes \Gamma$. For example, the second order term in (11) is then

$$\begin{aligned} U^{(2)}(+\infty,-\infty) &= -\frac{1}{2}\hat{T}\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \Big[:L_I(t)::L_I(t'): \\ &- \lambda_\alpha : \langle \bar{s}\gamma^\alpha s(t)\rangle :: L_I(t'): - :L_I(t): \lambda_\beta : \langle \bar{s}\gamma^\beta s(t')\rangle : \\ &+ \lambda_\alpha\lambda_\beta : \langle \bar{s}\gamma^\alpha s(t)\rangle :: \langle \bar{s}\gamma^\beta s(t')\rangle : \Big]. \end{aligned} \quad (13)$$

The second- and third-order terms, $U^{(2)}$ and $U^{(3)}$, are particularly interesting. Their contribution to the S-matrix, when written with the help of the interaction Hamiltonian H' and the sum over intermediate states $|k\rangle$,

$$\begin{aligned} S^{(2)}_{ab} &\equiv \langle b|U^{(2)}(+\infty,-\infty)|a\rangle = -2\pi i \delta(E_b - E_a) \\ &\times \sum_{k\neq a} \frac{\langle b| :H'(\lambda): |k\rangle\langle k| :H'(\lambda): |a\rangle}{E_a - E_k + i\epsilon}, \end{aligned} \quad (14)$$

$$
\begin{aligned}
S_{ab}^{(3)} &\equiv \langle b|U^{(3)}(+\infty,-\infty)|a\rangle = -2\pi i\delta(E_b - E_a) \\
&\quad \times \sum_{k,l\neq a} \frac{\langle b|:H'(\lambda):|k\rangle\langle k|:H'(\lambda):|l\rangle\langle l|:H'(\lambda):|a\rangle}{(E_a - E_k + i\epsilon)(E_a - E_l + i\epsilon)}\,,
\end{aligned} \tag{15}
$$

can obviously be related to $\Delta^{(2)}E_N$ and $\Delta^{(3)}E_N$ in (10) when $|a\rangle = |b\rangle = |N_0\rangle$:

$$
\langle N_0|U^{(i)}(+\infty,-\infty)|N_0\rangle = -2\pi i\delta(0)\Delta^{(i)}E_N(\lambda)\,, \qquad i = 2,3. \tag{16}
$$

[Strictly speaking, the divergence due to $\delta(0)$ renders this expression meaningless; however, we will be able to get rid of $\delta(0)$.] On the other hand, by using the standard field theory-expansion of U, *i.e.*, (11), we avoid the need to consider the intermediate states $|k\rangle$. To demonstrate this, let us for a moment concentrate on the contribution to strangeness that comes from $U^{(2)}(+\infty,-\infty)$, given by (13):

$$
\begin{aligned}
&\frac{\partial}{\partial\lambda_\mu}\langle N_0|U^{(2)}(+\infty,-\infty)|N_0\rangle\Big|_{\lambda_\mu=0} = \\
&= \langle N_0|\frac{1}{2}\hat{T}\int_{-\infty}^{+\infty} dt\int_{-\infty}^{+\infty} dt'\Big[\,:\langle\overline{s}\gamma^\mu s(t)\rangle::L_I(t'): \\
&\qquad\qquad + :L_I(t)::\langle\overline{s}\gamma^\mu s(t')\rangle:\,\Big]|N_0\rangle \\
&= \langle N_0|\int_{-\infty}^{+\infty} dt\Big\{\int_{-\infty}^{t} :\langle\overline{s}\gamma^\mu s(t)\rangle::L_I(t'):\,dt' \\
&\qquad\qquad + \int_{t}^{+\infty} :L_I(t')::\langle\overline{s}\gamma^\mu s(t)\rangle:\,dt'\Big\}|N_0\rangle\,.
\end{aligned} \tag{17}
$$

Since the nucleon strangeness cannot depend on the chosen time-slice t, the expression in the curly brackets must be the same for any t. We can therefore fix t in the curly brackets (*i.e.* in the limits of integration and in $\langle\bar{s}\gamma^\mu s(t)\rangle$) to any constant value $t = t_0$ (say, $t_0 = 0$), and we are free to factor out the expression in the curly brackets out of the integral over t:

$$
\begin{aligned}
&\frac{\partial}{\partial\lambda_\mu}\langle N_0|U^{(2)}(+\infty,-\infty)|N_0\rangle\Big|_{\lambda_\mu=0} = \left(\int_{-\infty}^{\infty} dt\right)\langle N_0|\Big\{ \\
&\times\int_{-\infty}^{t_0} :\langle\overline{s}\gamma^\mu s(t_0)\rangle::L_I(t'):\,dt' + \int_{t_0}^{+\infty} :L_I(t')::\langle\overline{s}\gamma^\mu s(t_0)\rangle:\,dt'\Big\}|N_0\rangle\,.
\end{aligned} \tag{18}
$$

This then leaves the integral ($\int_{-\infty}^{+\infty} dt$) as a constant but divergent prefactor. However, it exactly matches the constant divergent prefactor in (16), $2\pi\delta(0) = \int_{-\infty}^{\infty} dt$, and they cancel each other out. The inspection of the (18), (16), (10) and (9) then gives the contribution of $U^{(2)}$ to the nucleon strangeness. This is the first term on the right-hand side of (19) below (where we have again gathered the time-ordered integrals into one from $-\infty$ to $+\infty$ but containing the time-ordering operator $\hat{T}$). Repeating the above procedure for $U^{(3)}$ gives us the second term in (19). *I.e.*, the strange nucleon matrix element of the full nucleon state is then given by

$$\langle N| : \overline{s}\Gamma s : |N\rangle \;=\; i \int_{-\infty}^{+\infty} dt' \, \langle N_0|\hat{T} : \langle \overline{s}\Gamma s(t_0)\rangle : : L_I(t') : |N_0\rangle$$

$$- \; \frac{1}{2} \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dt'' \, \langle N_0|\hat{T} : \langle \overline{s}\Gamma s(t_0)\rangle : : L_I(t') : : L_I(t'') : |N_0\rangle \quad (19)$$

(We have reverted from the special case of γ_μ to a general matrix Γ.)

Obviously, the non-vanishing contributions to (19) occur only when the strange quark fields are fully contracted. *E.g.*, the integrand of the first term in (19), written in terms of space integrals over the strange current and Lagrangian densities is

$$\int d^3x \, d^3x' \langle N_0|\hat{T} : \overline{s}(x)\Gamma s(x) : : \mathcal{L}_I(x') : |N_0\rangle$$

$$= \quad \int d^3x \, d^3x' \langle N_0| : \overbrace{\overline{s}(x)\Gamma \underbrace{s(x)\mathcal{L}_I}}(x') : |N_0\rangle \; , \qquad (20)$$

(where the contractions are indicated by over- and underbraces, and $t_0 \equiv x_0, t' \equiv x'_0$ for consistency of the notation). So, the first term in (19) corresponds to Fig. 1, since these contractions, or time-ordered pairings, are of course the propagators of strange quarks. In the second term, the two contractions must connect the strangeness source at Γ with two different separately normal-ordered interaction Lagrangian densities which act as "sinks" for strangeness at two different points of a valence quark line, or two different valence quark lines. In any case, there must be an additional strange quark contraction between these two $: \mathcal{L}_I :$'s, and this completes the strange quark loop. Fig. 2 gives an example of the graphs originating from the second term of (19), namely the $U^{(3)}$ contribution. Clearly, this way kaon loops can be generated. If the result of [45] on small contribution of kaon loops is not an artifact of their model, it is likely that the second term in (19) will be much smaller than the first one if (19) is evaluated in realistic enough models. However, this cannot be known in advance. So, why not include still higher contributions which would give contributions like Fig. 3 for example?

First, let us remember that there are some perturbative interactions $\mathcal{L}_I$ which can nevertheless be important for nucleon strangeness. One example

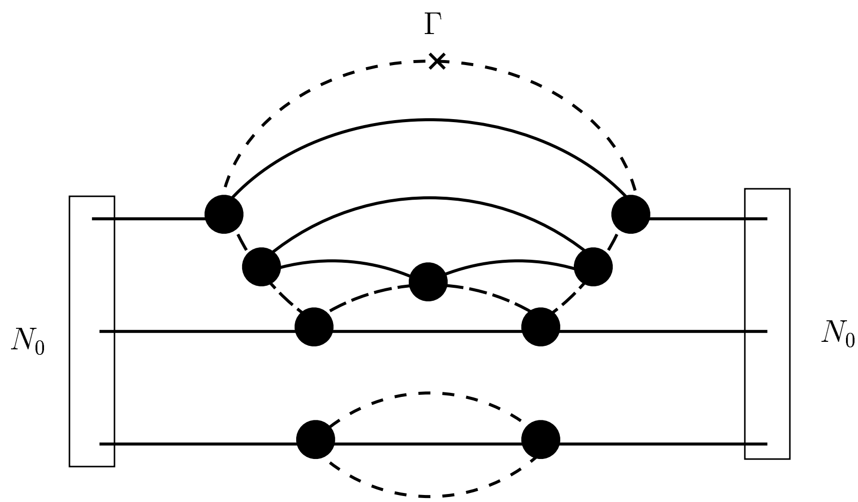

Figure 3: One of the spurious graphs that would originate from the tenth-order term $U^{(10)}$ of the evolution operator. It illustrates how the contributions from the terms higher than $U^{(3)}$ cannot be identified with responses to a strangeness source at Γ, but instead lead to dressing of already dressed quarks and to changing of the defined model interactions. As in the previous figures, black dots denote the interactions $\mathcal{L}_I$, solid lines denote nonstrange dressed quarks, and dashed lines strange dressed quarks.

is the depletion of the instanton density which occurs [58] in the MIT bag model, and which is so strong that in this case one can for sure treat the instanton-induced interactions perturbatively, using the density of very dilute instantons as the expansion parameter. In the light of results of [45] which indicate that instanton-induced interactions may be an especially important source of strangeness, even such a perturbative contribution from instanton-induced interactions may well be significant.

So, in some cases we can justify ignoring the contributions of $U^{(n)}, n \geq 4$, ($i.e$, $O[H'(\lambda)^4]$ in (10)) as a perturbative approximation[3]. However, we are actually better off than that, because our model prediction for strangeness terminates with the $U^{(3)}$ contribution even for nonperturbative interactions $\mathcal{L}_I$, because it turns out that contributions from higher $U^{(n)}$'s would be double-counting. This follows from our view on nucleon strangeness as the response (to a strangeness-sensitive probe) of nucleon model states which, in the static regime (before or after any interactions with external probes), are just $|N_0\rangle$, *i.e.*, are by assumption built only of non-strange *dressed* quarks

[3]Had we limited ourselves to such cases, we could have omitted most of our discussion of the role of dressed quarks in Secs. 1 and 2, because we would not have needed such detailed explanations for justification of the formula (19).

$\mathcal{U}, \mathcal{D}$, which hide all the complexity of QCD – including strange fluctuating pairs – in their (modeled) dressing. Remember that except the interactions induced by the strangeness source, all fundamental interactions (including $\mathcal{L}_I$) and resulting fluctuations of gluon *and* quark fields are lumped in forming these effective quarks and their effective model interactions and/or mean field they feel. Now, the contribution to (19) from a λ-differentiated term $U^{(n)}, n \geq 4$ (through (17)) would correspond to graphs with one vertex at Γ from which two propagators of dressed strange quarks would emanate, two $\mathcal{L}_I$-vertices which would receive them (so it would be like in Fig. 2, from the λ-differentiated $U^{(3)}$ contribution, so far), but then other $n-3$ $\mathcal{L}_I$-vertices would follow. (*E.g.*, see Fig. 3.) Depending on how the contractions are arranged, they can be connected in the loop originating at Γ, or can be disconnected from it, forming their separate loops. In both cases this is obviously double counting, as these additional $n-3$ vertices represent dressing of quarks that have already been dressed. The first two $\mathcal{L}_I$-vertices are different, as they are induced by the strangeness source — they are the unavoidable sink for the $s\bar{s}$ pair created by the source at Γ. The second term in (19), *i.e.*, the $U^{(3)}$ contribution, is the highest possible term that has just that, and does not contain additional interactions of the dressed strange loop with already dressed valence quarks, resulting in double-dressing. A completely equivalent, but probably even clearer way to see this is to view our external strangeness-sensitive probe as a *sink* (instead of a source) of a strange quark current. So, when this sink (by means of interaction $\mathcal{L}_I$) sucks a strange pair out of the polarization clouds that form our dressed valence quarks, this strange pair should go to this sink at Γ that pulled it out — and *not* run all around the nucleon interacting with the valence quarks up to $n-3$ times ($n = 4, 5, ..., \infty$), also altering in the process already defined model interactions between the valence quarks.

Realizing this also automatically answers why there is no such contractions among the *non*strange quark fields which would lead to additional $\mathcal{U}\bar{\mathcal{U}}$ and $\mathcal{D}\bar{\mathcal{D}}$ pairs. Such loops would also appear if contributions from higher $U^{(n)}$ could enter in (19). (See Fig. 3.) *I.e.*, the avoidance of double-dressing gives the response of $|N_0\rangle$ to a strangeness source in the generic form $|\delta N\rangle \sim |\mathcal{U}\mathcal{U}\mathcal{D}\mathcal{S}\bar{\mathcal{S}}\rangle$ and not $|\mathcal{U}\mathcal{U}\mathcal{D}\mathcal{S}\bar{\mathcal{S}}\mathcal{S}\bar{\mathcal{S}}\mathcal{D}\bar{\mathcal{D}}...\rangle$, *etc.*, without imposing by hand any additional limitations to "one-particle, one-hole" responses.

3 Strangeness evaluation with a specified interaction $\mathcal{L}_I$

Evaluation of the "master formula" (19) is in principle straightforward once one specifies two things: *i)* the overall description of the hadronic structure, which amounts to choosing the mean-field Hamiltonian H_0 in (5), and *ii)* $\mathcal{L}_I$, which generates the $q\bar{q}$ fluctuations. Namely, specifying *i)* should nor-

mally define also the single quark solutions; a concrete calculation within a specified framework or a model involves expanding of quark fields in an appropriate wave function basis (*e.g.*, in the next section we choose to employ the quark solutions for the MIT bag). The field contractions in (19) lead to the sums over stationary modes of single quarks and antiquarks, or, equivalently, the bound state propagators of these dressed model quarks. These sums require a regularization, but now, for the single quark modes, it is much easier to physically justify the choice of the cutoff than in the case of the sum over the exotic baryon and meson-baryon states like the one in (6) or in ref. [46]. Let us recall at this point that in the course of our derivation (7)–(20), we replaced these summation over hadronic states by summation over the states/modes of quarks which constitute these hadrons. The sum over quark modes should naturally run only up to some typical hadronic low-energy cutoff $\Lambda \sim 0.6$ GeV – 1 GeV. This cutoff on quark energies is dictated by the fact that nonperturbative interactions among quarks operate at low energies, whereas they gradually weaken and go over to the perturbative regime for higher energies. In the aforementioned study of $s\bar{s}$ effects of kaon loops [46], Geiger and Isgur have shown the importance of high-mass intermediate states in these loops. However, since these are hadronic, meson–baryon intermediate states, this does not contradict with cut-off such as $\Lambda \sim 1$ GeV on *quark* energies. Namely, the dominant portions of the results of Ref. [46] are accounted for by states lying below 3–3.5 GeV. For comparison, our cut-off of 1.1 GeV (see Table 1) imposed on the energies of one strange *quark* and one *antiquark* fluctuating on top of the valence nucleon state, corresponds to total energies up to $2\Lambda + M_N \sim 3$ GeV as well. This leads us to believe that we have accounted for the majority of important degrees of freedom in a way compatible with Ref. [46].

The cutoff values of 0.6 GeV – 1 GeV are typical for calculations in models of low-energy QCD, *e.g.*, the Nambu and Jona-Lasinio (NJL) model [45]. Obviously, we are supposing here that the nucleon strangeness is the effect of low-energy, nonperturbative QCD. Indeed, this brings us to the point *ii)*, *i.e.* to the question what to use concretely for $\mathcal{L}_I$ in (19) in the explicit calculation – performed in the next section – of $\langle N| : \overline{s}\Gamma s : |N\rangle$.

$\mathcal{L}_I$ can of course be any interaction which can produce fluctuating $s\overline{s}$ pairs, but the question is, which interactions can be important in producing the strangeness of the nucleon? For example, perturbative QCD interactions should be relatively unimportant in this regard. Although precisely the perturbative, high energy deep inelastic scattering reveals the sea of $q\overline{q}$ pairs, including $s\overline{s}$, the contribution of this perturbative sea to the nucleon strange matrix elements has traditionally been judged as relatively unimportant — see, *e.g.* Refs. [22, 38]. A theoretical analysis [64] of the CCFR data [65] on strange quark distribution functions from neutrino-nucleon deep inelastic scattering, seems to further support this point of view. For example, it finds a very small upper bound on the strange radius of the nucleon

($|\langle r^2\rangle_s| \leq 0.005$ fm^2) [64] when extracted from such parton distribution functions characterizing the nucleon structure at high momentum transfers. The possibly enhanced nucleon strangeness is thus expected (see *e.g.* [22]) as an effect of nonperturbative QCD which, at low energies, around nucleon mass scale, is certainly more important for hadronic structure than perturbative QCD, and can lead to $s\bar{s}$ pairs already at small momentum transfers, *i.e.*, large distances. Nonperturbative QCD is after all responsible for precisely such effects as forming quark-antiquark condensate $\langle 0|\bar{q}q|0\rangle$ ($q = u, d, s$) and gluon condensate characterizing the nonperturbative QCD vacuum. Some investigators (see, *e.g.*, ref. [66–68], or, for a recent and comprehensive review, ref. [69]) have suggested that among the most important nonperturbative configurations of the gluon fields are instantons. By now it is certainly well-established that the effective interaction between quarks resulting from the presence of instantons (let us call this interaction $\mathcal{L}_{inst}$), plays a very important role in the formation of hadron structure [69] although it is not responsible for confinement [70, 71], as thought previously. (In the present approach, confinement must anyway be taken care of by the unperturbed Hamiltonian H_0.) This $\mathcal{L}_{inst}$ is therefore in our opinion worth testing as an important candidate for the interactions $\mathcal{L}_I$ generating the strange nucleon matrix elements of some operators. A recent calculation [45] in the context of NJL model seems to be an indication that $\mathcal{L}_{inst}$ is indeed the most important part of $\mathcal{L}_I$, as it found large strange pair components in the nucleon only if instanton-induced interaction was included in the low-energy dynamics. In that case, the ratio y (2) can be several times larger than its upper limit in the case when the "standard" NJL model is used, even when augmented by kaon cloud effects [45].

Here we quote the vacuum-averaged version of the instanton-induced interaction $\mathcal{L}_{inst}$ derived by ref. [72] in the instanton liquid approach but transformed to x-space. It is actually convenient to separate it in one-, two-, and three-body pieces, $\mathcal{L}_1, \mathcal{L}_2$ and $\mathcal{L}_3$ respectively:

$$\mathcal{L}_{inst} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3, \tag{21}$$

$$\mathcal{L}_1 = -n \left(\tfrac{4\pi^2}{3}\rho^3 \right) \Big\{ \mathcal{F}_u \, \bar{u}_R \, u_L + (u \longleftrightarrow d) + (u \longleftrightarrow s) \Big\} + (R \longleftrightarrow L) \tag{22}$$

$$\mathcal{L}_2 = -n \left(\tfrac{4\pi^2}{3}\rho^3 \right)^2 \Big\{ \mathcal{F}_u \, \mathcal{F}_d \Big[(\bar{u}_R u_L)(\bar{d}_R d_L) + \frac{3}{32}(\bar{u}_R \lambda^a u_L \bar{d}_R \lambda^a d_L$$

$$- \frac{3}{4}\bar{u}_R \sigma_{\mu\nu} \lambda^a u_L \bar{d}_L \sigma^{\mu\nu} \lambda^a d_L) \Big] + (u \longleftrightarrow s) + (d \longleftrightarrow s) \Big\} + (R \longleftrightarrow L) \tag{23}$$

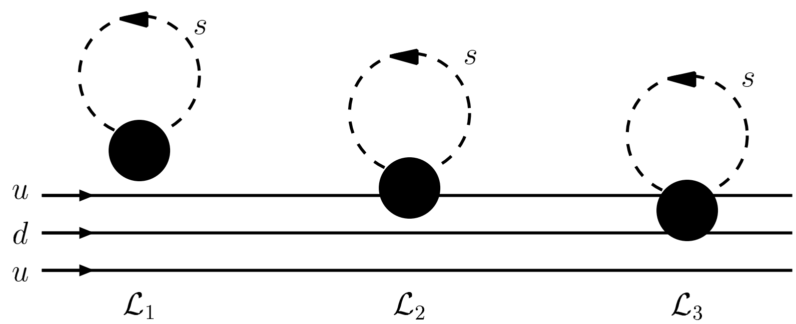

Figure 4: Instanton-induced local strangeness represented by the effective one-, two- and three-body operators. Non-strange quarks are denoted by solid lines, and strange ones by dashed lines.

$$\mathcal{L}_3 = -n\left(\ \frac{4\pi^2}{3}\rho^3\ \right)^3 \mathcal{F}_u\,\mathcal{F}_d\,\mathcal{F}_s\,\frac{1}{3!}\,\frac{1}{N_c(N_c^2-1)}\,\epsilon_{f_1f_2f_3}\,\epsilon_{g_1g_2g_3}\Bigg\{\frac{2N_c+1}{2N_c+4}$$
$$\times\,(\bar{q}_R^{f_1}q_L^{g_1})(\bar{q}_R^{f_2}q_L^{g_2})(\bar{q}_R^{f_3}q_L^{g_3}) + \frac{8}{3(N_c+3)}(\bar{q}_R^{f_1}q_L^{g_1})(\bar{q}_R^{f_2}\sigma_{\mu\nu}q_L^{g_2})(\bar{q}_R^{f_3}\sigma^{\mu\nu}q_L^{g_3})\Bigg\} \quad (24)$$

Here, n is the instanton density and $\mathcal{F}_f$'s are the characteristic factors (corresponding to inverse effective quark masses) composed of current light quark masses m_f $(f = u, d, s)$, average instanton size $\rho \simeq \frac{1}{3}$ fm [67, 68, 73], and the quark condensate $\langle 0|\overline{q}q|0\rangle = (-240\text{MeV})^3$. *E.g.*, for the u-flavour, $\mathcal{F}_u \equiv [m_u\rho - \frac{2\pi^2}{3}\rho^3\langle 0|\overline{q}q|0\rangle]^{-1}$, and analogously for the other flavours. The left (and right) projected components are defined in the usual way; *e.g.*, for the u-flavour, $u_{L,R} = \gamma_\pm u \equiv \frac{1}{2}(1 \pm \gamma_5)u$.

In the three body interaction $\mathcal{L}_3$, the indices f_i, g_i $(i = 1, 2, 3)$ run over light flavours u, d, and s. *E.g.*, $g_3 = d$ means $q_L^{g_3} = d_L$. Repeated indices are summed over. The interaction defined here by $\mathcal{L}_1, \mathcal{L}_2$, and $\mathcal{L}_3$ is actually the same as the well-known one of Shifman, Vainshtein and Zakharov (SVZ) [57], although the present three-body term (24) looks much simpler, but it is just that Nowak [74] Fierzed away otherwise very complicated color structures in that piece of SVZ interaction [57], reshuffling them in simple prefactors involving the number of quark colors N_c. Obviously, the two-body term is the one which, through the (19) and (20), yields the graph in Fig. 1. In addition to that, there is also a contribution to the nucleon strangeness due to the three-body interaction $\mathcal{L}_3$, exemplified by the last loop in Fig. 4. Such graphs come about when contractions in (20) are done with a strange bilinear in $\mathcal{L}_3$.

In contradistinction to $\mathcal{L}_2$ and $\mathcal{L}_3$, the contribution to the nucleon strangeness due to the one-body term $\mathcal{L}_1$ does not involve any interacting with

the valence quarks, as illustrated in Fig. 3. Perhaps not surprisingly, this disconnected graph requires some care. $\mathcal{L}_1$ has in fact the form of a mass term, and can be thought of as the self-energy, or the effective mass that a quark acquires from the effective interaction caused by the instanton liquid through which quarks move in the nonperturbative QCD vacuum. Now imagine that we want to evaluate some strange nucleon matrix element (19) in some kind of constituent quark model where one from the start uses effective constituent quark masses already "dressed" by nonperturbative QCD. The self-mass part of the instanton effects would in that case be already included in the constituent mass parameters. Using $\mathcal{L}_1$ in the calculation would therefore be double-counting, so in that case it must be dropped. On the other hand, if we would use some approach where one uses the current, Lagrangian quark masses, like in the MIT bag model for example, there is no reason to drop $\mathcal{L}_1$ and it should be included in the calculation on equal footing with $\mathcal{L}_2$ and $\mathcal{L}_3$.

We also note that the average instanton size $\rho \simeq \frac{1}{3}$ fm $= (600\text{MeV})^{-1}$ is consistent with what we said above about the typical hadronic cutoff scale $\Lambda \sim 0.6$–1 GeV. Namely, the effective interaction $\mathcal{L}_{inst}$ cannot be operative at energies which would probe distances significantly smaller than the average size of these extended objects, instantons, which produce $\mathcal{L}_{inst}$.

The final point we should clarify concerns consistency of using the instanton induced interaction $\mathcal{L}_{inst}$ for $\mathcal{L}_I$ in (19), even in the case when we view (19) as a purely perturbative result.

Namely, although in the previous section we have advanced the arguments why the applicability of (19) goes beyond perturbation theory in the context of some quark models, we want to point out that even if we forget for a moment these arguments, what we do in the next section can be justified already from a viewpoint that is essentially perturbative. So, if we take this viewpoint, why is (19) applicable not only to parts of $\mathcal{L}_I$ which come from perturbative interactions like the perturbative gluon exchange, but also to $\mathcal{L}_{inst}$ (21)–(24) which is of nonperturbative origin, precisely wherefrom its importance derives in this low-energy context? The point is that the *origin* of $\mathcal{L}_{inst}$ is nonperturbative, *i.e.*, these effective interactions between quarks are the consequence of nonperturbative gluon configurations — instantons. However, $\mathcal{L}_{inst}$ itself contains a small parameter, namely the instanton density n, and it is in fact so small that perturbative expansion in its powers is possible. Original estimates [73] where $n \approx 1.6 \cdot 10^{-3}$ GeV^4 proved to be quite reliable as they have remained essentially unchanged [69] also in the more recent instanton liquid calculations. It is in fact useful to define a "dimensionless instanton density" $\tilde{n}$ by expressing it in units of the average instanton size ρ, $n \equiv \tilde{n}\rho^{-4}$. The commonly accepted value is $\rho = 1/600$ MeV^{-1} $\simeq 1/3$ fm [67, 68, 75]. Therefore $\tilde{n} \simeq 12.4 \cdot 10^{-3} \simeq 1/81$ and this dimensionless parameter is obviously small enough to be used as the parameter of the perturbative expansion. (The expansion parameter in QED is

not much smaller, $\alpha \simeq 1/137$.) We should also keep in mind that this is the instanton density in the true, nonperturbative QCD vacuum, while in some circumstances the appropriate n can be even smaller. Notably, ref. [58] found that in the MIT bag model enlarged with the instanton–induced interaction (21)–(24), which is used in the next section for the first evaluations of the nucleon strangeness via formula (19), the instanton density is very strongly depleted with respect to the true QCD vacuum. Ref. [58] used certain approximations and assumptions, so that the depletion may be not quite so strong as estimated there, but the usage of $\mathcal{L}_{inst}$ in (19) is clearly consistent anyway, since even the aforementioned value of the undepleted instanton density in the truly nonperturbative QCD vacuum is small enough to serve as the parameter of the perturbative expansion.

4 Instanton–induced strangeness in the MIT bag model

Now, we turn to the actual calculation of strange nucleon matrix elements in the MIT bag model and with the instanton-induced interaction $\mathcal{L}_{\rm inst}$ given by (22-24). For definiteness, we quote the results for the proton—since the neutron case is quite similar, we keep $|N\rangle$ (for nucleons) in our expressions.

Using (19), the proton-strangeness matrix element is

$$\begin{aligned} \langle N| : \bar{s}\Gamma s : |N\rangle &= i \int_{-\infty}^{\infty} dt' \, \langle N_0| \hat{T} : \int d^3x \, \bar{s}(\boldsymbol{x}, t_0)\Gamma s(\boldsymbol{x}, t_0) : \\ &\times \; : \int d^3y \, \mathcal{L}_{\rm inst}(\boldsymbol{y}, t') : |N_0\rangle \,, \end{aligned} \tag{25}$$

where we have kept only the first term in the perturbation series over low instanton density. We have treated each of the three parts of $\mathcal{L}_{\rm inst}$ (21) separately. The one-body interaction $\mathcal{L}_1$ (22) is the simplest of all. Since no valence quarks take part in this interaction, the only relevant part of $\mathcal{L}_1$ is

$$-\, n \left(\frac{4\pi}{3}\rho^3\right) \mathcal{F}_s(\bar{s}_R s_L + \bar{s}_L s_R) \,, \tag{26}$$

giving the $\mathcal{L}_1$ part of the matrix element:

$$\begin{aligned} \langle N| : \bar{s}\Gamma s : |N\rangle_{\mathcal{L}_1} &= i \int_{-\infty}^{\infty} \langle N_0|N_0\rangle \, \hat{T} : \int d^3x \, \bar{s}(\boldsymbol{x}, t_0)\Gamma s(\boldsymbol{x}, t_0) : \\ &\times \; : \int d^3y \, \bar{s}(\boldsymbol{y}, t')s(\boldsymbol{y}, t') : \,. \end{aligned} \tag{27}$$

By taking into account the expansion of the strange quark quantum field $s(x)$ in the MIT bag-model wave functions $s_K(\boldsymbol{x})$ (see Appendix), this reduces

n	κ	$\omega_{n\kappa}$ /MeV
0	-1	514.0
0	-2	726.7
0	1	797.4
1	-1	1104.9

Table 1: Strange quark states which can be excited by the instanton interaction.

to the

$$\begin{aligned}\langle N| : \bar{s}\Gamma s : |N\rangle_{\mathcal{L}_1} &= 4\pi^2 n\rho^3 \mathcal{F}_s \sum_{K,L} \frac{1}{\omega_K + \omega_L} \\ &\times \left\{ \int d^3x\, \bar{s}_M(\boldsymbol{x})\Gamma s^c_N(\boldsymbol{x}) \int d^3y\, \bar{s}^c_N(\boldsymbol{y}) s_M(\boldsymbol{y}) \;+\; (s \leftrightarrow s^c) \right\} . \end{aligned} \tag{28}$$

Here, K and L stand for sets of quantum numbers labelling quark states in the bag $K = \{n, \kappa, j_3\}$, $L = \{n', \kappa', j'_3\}$ (see Appendix). The sum over K and L goes up to the state with $n = 1$, $\kappa = -1$ (corresponding to the cut-off of about 1.1 GeV), encompassing four lowest-lying strange quark states displayed in Table 1.

The expression for the contribution of the two-body interaction $\mathcal{L}_2$ is somewhat more complicated, involving also valence quark wave functions. Luckily, the terms with $\sigma^{\mu\nu}$ cancel out, leaving us with

$$\begin{aligned}\langle N| : \bar{s}\Gamma s : |N\rangle_{\mathcal{L}_2} &= \frac{16}{3}\pi^4 n\rho^6 \mathcal{F}_q \mathcal{F}_s \sum_{K,L,\pm} \frac{1}{\omega_K + \omega_L} \\ \times \Bigg\{ &\int d^3x\, \bar{s}_K(\boldsymbol{x})\Gamma s^c_L(\boldsymbol{x}) \int d^3y\, \bar{s}^c_L(\boldsymbol{y})\gamma_\pm s_K(\boldsymbol{y}) \\ &\times \left[2\bar{q}_{0,-1,\frac{1}{2}}(\boldsymbol{y})\,\gamma_\pm\, q_{0,-1,\frac{1}{2}}(\boldsymbol{y}) \;+\; \bar{q}_{0,-1,-\frac{1}{2}}(\boldsymbol{y})\,\gamma_\pm\, q_{0,-1,-\frac{1}{2}}(\boldsymbol{y}) \right] \\ + &\int d^3x\, \bar{s}^c_K(\boldsymbol{x})\Gamma s_L(\boldsymbol{x}) \int d^3y\, \bar{s}_L(\boldsymbol{y})\gamma_\pm s^c_K(\boldsymbol{y}) \\ &\times \left[2\bar{q}_{0,-1,\frac{1}{2}}(\boldsymbol{y})\,\gamma_\pm\, q_{0,-1,\frac{1}{2}}(\boldsymbol{y}) \;+\; \bar{q}_{0,-1,-\frac{1}{2}}(\boldsymbol{y})\,\gamma_\pm\, q_{0,-1,-\frac{1}{2}}(\boldsymbol{y}) \right] \Bigg\} \end{aligned} \tag{29}$$

Here $q_{0,-1,\pm\frac{1}{2}}(\boldsymbol{y})$ is the wave function for the ground state of the valence quark in the bag, which we take to be the same for u and d quarks.

Going now to the three-body interaction $\mathcal{L}_3$, expressions become extremely long and complicated, so we do not write them down here. In any case, as seen below, it turns out that this contribution is much smaller than the preceding two.

After focusing on the scalar ($\bar{s}s$) and pseudoscalar ($\bar{s}\gamma_5 s$) strangeness as the channels preferred by the QCD-vacuum fluctuations [76] we have checked the vector ($\bar{s}\gamma_\mu s$) and the axial-vector ($\bar{s}\gamma_\mu\gamma_5 s$) channels too.

The calculation of the contribution of the two-body, $\mathcal{L}_2$, and three-body, $\mathcal{L}_3$, instanton interactions is tedious and in manipulation of all these formulae we have relied heavily on *Mathematica* package [77] for symbolic computer calculations.

To get a rough idea how the calculation in the MIT bag model was performed and in which way such a model choice influences our results, we briefly sketch the calculation with the one-body part $\mathcal{L}_1$ interaction bellow.

4.1 Scalar and pseudoscalar strangeness

Let us first consider the **scalar** strange current density $\bar{s}s$ inside the proton. The expression for the matrix element can be written as

$$\langle N(p')|\bar{s}s|N(p)\rangle = A_s(q^2)\bar{u}_N(p')u_N(p) \ , \tag{30}$$

where $q^2 = (p-p')^2$, and u_N's are nucleon spinors. $A_s(q^2)$ is the scalar form factor accounting at $q^2 = 0$ for the scalar strangeness of the proton.

Calculations inside the bag model can be performed by making the substitution $\Gamma = 1$ and inserting the appropriate quark and antiquark wave functions in (28). By a simple calculation one can show that the surviving combination is the one with $\kappa = -1$, $\kappa' = 1$ and $\kappa = 1$, $\kappa' = -1$. Therefore,

$$\begin{aligned} \langle N| : \bar{s}s : |N\rangle_{\mathcal{L}_1} &= 4\pi^2 n\rho^3\mathcal{F}_s \sideset{}{'}\sum_{K,L,\kappa,\kappa'=-1,1} \frac{1}{\omega_K+\omega_L} \\ &\times \left\{\int d^3x\, \bar{s}_K(\boldsymbol{x})s^c_L(\boldsymbol{x}) \int d^3y\, \bar{s}^c_L(\boldsymbol{y})s_K(\boldsymbol{y}) \ + \ (s \leftrightarrow s^c)\right\} \ , \end{aligned} \tag{31}$$

where $\sum'$ denotes the incomplete sum where the cases with equal κ quantum numbers are omitted, and

$$\begin{aligned} \langle N| : \bar{s}s : |N\rangle_{\mathcal{L}_1} &= 4\pi^2 n\rho^3\mathcal{F}_s \sum_{n=0}^{1} 4 \\ &\times \Big[2N_{-1}(x_{n,-1})N_1(x_{0,1}) \int r^2 dr W_+(n,-1)W_-(0,1)j_0(x_{n,-1}\frac{r}{R})j_0(x_{0,1}\frac{r}{R}) \\ &+W_-(n,-1)W_+(0,1)j_1(x_{n,-1}\frac{r}{R})j_1(x_{0,1}\frac{r}{R})\Big]^2 \ . \end{aligned} \tag{32}$$

The normalizations $N_{\pm1}(x_{n,\pm1})$ and the $W_\pm$-factors are given in the Appendix. The above equation represents the contribution to the strange scalar form factor $A_s(q^2 = 0)$ coming from the one-body interaction. The remaining contributions from the $\mathcal{L}_2$ and $\mathcal{L}_3$ instanton interactions can be calculated similarly and the results are

$$\langle N| : \bar{s}s : |N\rangle_{\mathcal{L}_1} = 0.035 \ , \tag{33}$$

$$\langle N| : \bar{s}s : |N\rangle_{\mathcal{L}_2} = 0.023 \ , \tag{34}$$

$$\langle N| : \bar{s}s : |N\rangle_{\mathcal{L}_3} = 2.9 \cdot 10^{-4} \ . \tag{35}$$

Summing them up gives

$$A_s(0)_{\mathcal{L}_{\rm inst}} = 0.058. \tag{36}$$

The evaluation of space-integrals was performed numerically, using the following values for the parameters: the bag radius R=1/197.3 MeV^{-1} ≈1 fm, the average instanton size ρ=1/600 MeV^{-1}, and the instanton density $n = 2.66 \cdot 10^7$ MeV^4, which is depleted instanton density in the MIT bag as found in [58]. Moreover, we take the strange quark mass m_s=200 MeV and the valence quark mass $m_u = m_d \equiv m_q$=8 MeV. The quark condensate that follows from the Gell-Mann–Oakes–Renner relation for these quark masses and the empirical meson masses is $\langle 0|\bar{q}q|0\rangle \approx (-200\text{MeV})^3$.

The **pseudoscalar** strange form factor B_s is defined as

$$\langle N(p')|\bar{s}\gamma_5 s e^{-i\boldsymbol{q}\cdot\boldsymbol{x}}|N(p)\rangle = B_s(q^2)\bar{u}_N(p')\gamma_5 u_N(p)\ . \tag{37}$$

For the pseudoscalar strange current $\bar{s}\gamma_5 s$, (28) gives the vanishing one-body contribution

$$\langle N| : \bar{s}\gamma_5 s : |N\rangle_{\mathcal{L}_1} = 0\ . \tag{38}$$

Analogously, we obtain the vanishing result for the other two instanton interactions, i.e. $\langle N| : \bar{s}\gamma_5 s : |N\rangle_{\mathcal{L}_{\rm inst}} = 0$. We thus obtain

$$B_s(0)_{\mathcal{L}_{\rm inst}} = 0\ , \tag{39}$$

as the vanishing total instanton contribution to the pseudoscalar form factor.

4.2 Vector and axial-vector strangeness

Recently, there has been a lot of experimental activity [78, 79] devoted to the **vector** strangeness, described by Dirac (F_1) and Pauli (F_2) form factors in the decomposition

$$\langle N|\bar{s}\gamma_\mu s|N\rangle = \bar{u}_N(p')\left[F_1^s(q^2)\gamma_\mu + F_2^s(q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_N}\right]u_N(p)\ . \tag{40}$$

For the comparison with the experimental data, the Sachs form factors, G_E (electric) and G_M (magnetic) are widely used:

$$\begin{aligned} G_E(q^2) &= F_1(q^2) + \frac{q^2}{4M_N^2}F_2(q^2)\,, \\ G_M(q^2) &= F_1(q^2) + F_2(q^2)\,, \end{aligned} \tag{41}$$

with definitions $eG_E^{(0)} = Q$ (charge) and $(e/2M_N)G_M^{(0)} = \mu$ (magnetic moment). By taking the non-relativistic nucleon spinor

$$u_N(p,s) = \sqrt{\frac{E+M_N}{2E}}\begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s \end{pmatrix} \tag{42}$$

in the Breit frame, defined by

$$
\begin{aligned}
q^\mu &= (q^0, \boldsymbol{q}) = (0, \boldsymbol{q}_B) \,, \\
\boldsymbol{p} &= \frac{\boldsymbol{q}_B}{2} \,, \; \boldsymbol{p'} = -\frac{\boldsymbol{q}_B}{2} \,,
\end{aligned} \tag{43}
$$

the components of the vector current receive the form

$$
\begin{aligned}
\langle N(p', s')|V_0^s|N(p,s)\rangle &= \frac{m}{E}\chi^\dagger_{s'}\chi_s G_E^{(s)}(-q_B^2) \,, \\
\langle N(p', s')|\boldsymbol{V}^s|N(p,s)\rangle &= \frac{1}{2E}\chi^\dagger_{s'} i(\boldsymbol{\sigma}\times\boldsymbol{q}_B)\chi_s G_M^{(s)}(-q_B^2) \,.
\end{aligned} \tag{44}
$$

In order to calculate the contribution from the instanton induced vector strange current inside the MIT bag, we have to identify the form factors in (44) with the Fourier transformed vector current within the bag

$$
\langle N(p')| :V_\mu^s(q^2): |N(p)\rangle_{\mathcal{L}_{\rm inst}} = \langle N(p')| :\int d^3\boldsymbol{r} e^{-i\boldsymbol{q}_B\cdot\boldsymbol{r}}\bar{s}(\boldsymbol{r})\gamma_\mu s(\boldsymbol{r}) : |N(p)\rangle_{\mathcal{L}_{\rm inst}}, \tag{45}
$$

in the static limit $q \to 0$. Simple check with $V_0^s(q^2)$ component of the vector current gives zero, *i.e.* $G_E^{(s)}(q^2=0)_{\rm inst} = 0$ as it should be.

A similar calculation for the space components $\boldsymbol{V}^s$ shows a non-trivial cancellation among the contributions of quarks in the loop with different spin orientations, producing the total result

$$
G_M^s(0)_{\mathcal{L}_{\rm inst}} = 0 \,. \tag{46}
$$

This implies the vanishing strange magnetic moment

$$
\mu_s = F_2^s(0) = 0 \,, \tag{47}
$$

which is compatible with the recent measurements at MIT/Bates [78] and even more recent ones at TJNAF (JLab) [79].

The estimation of the **axial-vector** strangeness can be done along the same lines. The form-factor decomposition, assuming the G-parity symmetry of strong interactions, has the form

$$
\begin{aligned}
&\langle N(p')|\bar{s}\gamma_\mu\gamma_5 s|N(p)\rangle \\
&\quad = \bar{u}_N(p')\left(\gamma_\mu\gamma_5 G_1^s(q^2) + \frac{q_\mu}{2M_N}\gamma_5 G_2^s(q^2)\right)\bar{u}_N(p) \,.
\end{aligned} \tag{48}
$$

The instanton contribution to such a matrix element can be calculated as

$$
\begin{aligned}
&\langle N(p')| : A_\mu^s : |N(p)\rangle_{\mathcal{L}_{\rm inst}} \\
&\quad = \langle N(p')| : \int d^3r e^{-i\boldsymbol{q}_B\cdot\boldsymbol{r}}\bar{s}(\boldsymbol{r})\gamma_\mu\gamma_5 s(\boldsymbol{r}) : |N(p)\rangle_{\mathcal{L}_{\rm inst}}
\end{aligned} \tag{49}
$$

and should be compared with the axial form factors defined in the Breit frame as

$$\langle N(p', s')|\boldsymbol{A}^s|N(p, s)\rangle = G_A^s(0)\chi_{s'}^\dagger \boldsymbol{\sigma}\chi_s \ . \tag{50}$$

Again, it turns out that the axial-vector strangeness induced by the instanton interaction is vanishing,

$$G_A^s(0)_{\mathcal{L}_{\rm inst}} = 0 \ . \tag{51}$$

5 Discussion and conclusions

The original MIT bag model [80–82] represents a suitable starting point in predicting the low-energy properties of low-mass hadrons. In this model, $R_{\rm bag}$ corresponds to the separations $R_{\rm confining} \sim 1$ fm at which confinement effects are important, arising at the scale $\Lambda_{QCD} \simeq 100$ to 300 MeV. Short–distance effects are supposedly taken care of by the perturbative one–gluon exchange.

However, in order to account for the effects at intermediate distances, *i.e.* at the momentum scales $Q \sim \Lambda_{\chi SB} \simeq 0.6$ GeV, the effective interaction (21)-(24), induced by the liquid of small instantons (of the average size $\rho = 1/3$ fm) appears appropriate. Of course, the effects of the instanton–induced interactions are not included in Donoghue and Nappi's [1] simple bag-model relation

$$\langle N|\bar{s}s|N\rangle = -\langle 0|\bar{s}s|0\rangle V \tag{52}$$

for the scalar nucleon strangeness, and the relative importance of this naive strangeness and the instanton effects is precisely what interests us here.

An advantage of the formula (19) is that in principle it treats the scalar, pseudoscalar, vector, axial, tensor or pseudotensor nucleon strangeness in a unified manner; one just has to specify what Γ is. Within a chosen nucleon model, the evaluation of (19) would proceed in — essentially — the same way for each Γ, except for technical details. Nevertheless, these technical details make a huge difference in practice because, as clarified in the previous section, even if the scalar and pseudoscalar cases are tractable, the tensor or pseudotensor cases seem prohibitively hard to do. However, this is a significant difference only now, at the present capabilities of symbolic manipulation software, and will diminish with the certain advancement of this software and computer power in the future.

In the scalar case ($\Gamma = 1$), the naive bag-model strangeness (52) is actually rather large for standard values of parameters. For our values, given at the end of subsection 4.1, it is

$$A_s^{\rm Nbag} \equiv -\langle 0|\bar{q}q|0\rangle V_{\rm bag} = 4.36 \ , \tag{53}$$

which is much larger than the instanton-induced contribution (36), and dominates the summed strangeness

$$A_s \equiv A_s^{\mathrm{Nbag}} + A_s(0)_{\mathcal{L}_{\mathrm{inst}}} = 4.42 \ . \tag{54}$$

Owing to using a somewhat smaller value of the quark condensate, Donoghue and Nappi [1] obtained 3.6 for this naive strangeness, which is still rather large. A_s^{Nbag} depends very strongly on the model size parameter R_{bag} since $V_{\mathrm{bag}} = R_{\mathrm{bag}}^3 4\pi/3$. For example, A_s^{Nbag} would decrease by a factor of 2 if $R_{\mathrm{bag}} = 0.8$ fm, a nucleon size which may be more acceptable, as the standard MIT bag value of 1 fm seems too large (*e.g.*, see [83]). However, since the model dependence on the bag radius is similar for other presently interesting matrix elements, the model dependence largely cancels out when one forms ratios. In particular, the instanton-induced contribution (36) remains small in comparison with the naive nucleon bag strangeness,

$$\frac{A_s^{\mathrm{Nbag}}}{A_s(0)_{\mathcal{L}_{\mathrm{inst}}}} \sim 75 \ , \tag{55}$$

for reasonable variations of the radius parameter.

Note that using $\mathcal{L}_{inst}$ for $\mathcal{L}_I$ in (19) enables one to see what happens in different models with the intriguing results of Steininger and Weise [45] concerning the importance of the instanton-induced interaction for the scalar strangeness of nucleons. Our results in the MIT bag model happen to disagree with their results in the NJL model enlarged with 't Hooft's instanton-induced interaction. Our results indicate that the instanton-induced interaction contributes just a small fraction to the – otherwise rather large [1] – scalar strangeness of nucleons modeled as MIT bags.

Obviously, the contribution due to the difference in the condensate with respect to the true, non-perturbative QCD vacuum dominates the strangeness in the nucleon bag. Admittedly, the instanton-induced contribution of this size *would* be obtained in the calculation of (36) *if* one would — inside the MIT bag — use the non-depleted instanton density $n = 1.6 \cdot 10^9$ MeV^4. However, we consider this merely as a consistency check, and not as an alternative description of strangeness in the MIT bag, because using the instanton density appropriate to the non-perturbative QCD vacuum containing the large quark condensate, would imply assuming the nonperturbative QCD vacuum and the quark condensate not only outside, but also inside the bag. This would indeed enable $A_s(0)_{\mathcal{L}_{\mathrm{inst}}}$ to replace A_s^{Nbag} in full, but would also make the MIT bag description inconsistent [58].

The diluteness of the instanton liquid justifies the one-instanton approximation (*i.e.*, the first order in the perturbation theory for $\mathcal{L}_{inst}$) indicated in Fig. 4. The second–order contributions to (19) should be even smaller than the small first–order results on instanton–induced strangeness we obtained in the MIT bag model. This removes the motivation for evaluating them, at

least in the framework of that model. Of course, in some other models, and possibly also with some other $\mathcal{L}_I$, the results on the nucleon strangeness it induces can be considerably higher, making the evaluation of the second–order contributions more interesting. As commented above, if one would find in different models that the second term in (19) is small in comparison with the first term in (19), one would corroborate the result of [45] that virtual kaon loops contribute little to the scalar strangeness. For the reasons explained above, this conclusion indeed seems natural in the present approach. More generally, our result (19) may well help to clarify the relationship (which, *e.g.*, [49] judges as rather unclear) between the kaon loop contribution, and the ϕ-meson pole contribution [13], or the vector-meson (ϕ, ω) dominance contribution [50]. Namely, we believe that it will be possible (for $\Gamma = \gamma_\mu$) to relate the first term in (19) to such ϕ-meson contributions in a way similar to the relationship of the second term in (19) with the kaon loop contribution. More recent evaluation [84] based on the up-to-dated information on the (soft) nucleon-hyperon-K^* form factors yields the results reduced by more than an order of magnitude. This brings the vector strangeness closer to our result (46), which is compatible with the recent measurements.

The scalar strangeness is special because of non-vanishing scalar $q\bar{q}$ condensates of the QCD vacuum, which makes it more natural that it is larger than vector, axial or other strangeness channels. This is especially clear in our approach applied to the MIT bag model, where the scalar strangeness comes mostly from the difference of the scalar $q\bar{q}$ condensates in the true QCD vacuum and their absence in the perturbative vacuum inside the cavity [1], while only the relatively small remainder comes from the response of the valence ground state to the strangeness–sensitive probe. However, such a response is all that exists in the case of the pseudoscalar, vector, axial, *etc.*, nucleon strangeness, since there are no pseudoscalar, vector, axial, *etc.*, QCD-vacuum condensates either inside or outside the cavity. Since such responses tend to be much smaller than the nonperturbative vacuum contributions, significant differences in magnitude between the scalar and other kinds of strangeness are very natural in our approach. In fact, in the present case of the MIT bag model, we find the vanishing first–order contribution to the vector strangeness. The vanishing first–order contributions are found also for the pseudoscalar and axial strangeness of the nucleon.

This confirms the conjecture of ref. [76] for the case of the scalar strangeness. Our results are also consistent with the most recent measurements of the strange vector form factors at low momentum transfer, $Q^2 \lesssim 1$ GeV. The experimental strange magnetic form factor of the nucleon at $Q^2 = 0.1$ $(\mathrm{GeV/c})^2$, $G_M^s = 0.23 \pm 0.37 \pm 0.15 \pm 0.19\,\mu_N$, obtained at MIT/Bates [78] is consistent with the absence of strange quarks, but the error bars are large. However, the results and conclusions of our approach, that channels other than the scalar one should not be appreciably affected by strange

quarks, seems to get support especially from the most recent and very precise TJNAF (JLab) measurement [79] yielding the small strange vector form factors at $Q^2 = 0.48$ (GeV/c)^2, $G_E^s + 0.39 G_M^s = 0.023 \pm 0.034 \pm 0.022 \pm 0.026\,\mu_N$

This also makes understandable why the results on the "non-scalar" strange quantities such as the strangeness nucleon magnetic form factor [13, 26, 48, 51–56] or the strangeness electric mean-square radius [13, 26, 46, 48, 50–56] vary so much, even by the sign, from one model to another: the "non-scalar" strange quantities should all be rather small, and artifacts of various models very easily put in on either side of the zero.

Appendix: MIT bag–model wave functions

Quantum fields for quarks of flavour $q = u, d$ or s in the MIT bag model are

$$q(x) = \sum_K \left(\mathcal{Q}_K\, q_K(\boldsymbol{r}) e^{-i\omega_K t} + \mathcal{Q}_K^{c\dagger}\, q_K^c(\boldsymbol{r}) e^{i\omega_K t} \right) \,, \tag{A1}$$

$$\bar{q}(x) = \sum_K \left(\mathcal{Q}_K^{\dagger}\, \bar{q}_K(\boldsymbol{r}) e^{i\omega_K t} + \mathcal{Q}_K^{c}\, \bar{q}_K^c(\boldsymbol{r}) e^{-i\omega_K t} \right) \,, \tag{A2}$$

where $\mathcal{Q}, \mathcal{Q}^\dagger, \mathcal{Q}^c$ and $\mathcal{Q}^{c\dagger}$ are annihilation and creation operators for quarks and antiquarks, respectively. Quark and antiquark wave functions, specified by the quantum numbers $K = \{n, j, j_3, l\}$, are [85]

$$q_{njj_3l}(\boldsymbol{r}) = N_{jl}(x_{njl}) \begin{pmatrix} iW_+(njl)\; j_l\left(x_{njl}\frac{r}{R}\right)\; \phi_{jj_3l}(\hat{\boldsymbol{r}}) \\ \\ (\bar{l}-l)W_-(njl)\; j_{\bar{l}}\left(x_{njl}\frac{r}{R}\right)\; \phi_{jj_3\bar{l}}(\hat{\boldsymbol{r}}) \end{pmatrix} \,, \tag{A3}$$

$$q^c_{njj_3l}(\boldsymbol{r}) = N_{jl}(x_{njl}) \begin{pmatrix} iW_-(njl)\; j_{\bar{l}}\left(x_{njl}\frac{r}{R}\right)\; \phi_{jj_3\bar{l}}(\hat{\boldsymbol{r}}) \\ \\ (\bar{l}-l)W_+(njl)\; j_l\left(x_{njl}\frac{r}{R}\right)\; \phi_{jj_3l}(\hat{\boldsymbol{r}}) \end{pmatrix} \,. \tag{A4}$$

Here

$$\bar{l} = j \mp \frac{1}{2} \quad \text{when} \quad l = j \pm \frac{1}{2} \,, \tag{A5}$$

and

$$W_\pm(njl) = \sqrt{\frac{\omega_{njl} \pm m_q}{\omega_{njl}}} \,. \tag{A6}$$

The normalization constant is

$$N_{jl}^{-2}(x_{njl}) = \frac{R^3 j_l^2(x_{njl})}{\omega_{njl}(\omega_{njl} - m_q)} \left\{ 2\omega_{njl} \left[\omega_{njl} - (\bar{l}-l)\frac{j+\frac{1}{2}}{R} \right] + \frac{m_q}{R} \right\} \,, \tag{A7}$$

and the angular parts of the wave functions are

$$\phi_{jj_3l}(\hat{\boldsymbol{r}}) = \sum_{l_3 s_3} \langle jj_3|ll_3, \frac{1}{2}s_3\rangle \, Y_l^{l_3}(\hat{\boldsymbol{r}}) \, \chi_{s_3} \ , \tag{A8}$$

$$\phi_{jj_3\bar{l}}(\hat{\boldsymbol{r}}) = -\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}\phi_{jj_3l}(\hat{\boldsymbol{r}}) \ . \tag{A9}$$

Here, $Y_l^{l_3}$ are spherical harmonics, χ_{s_3} are Pauli spinors, $\langle jj_3|ll_3, \frac{1}{2}s_3\rangle$ are Clebsch-Gordan coefficients, $\boldsymbol{\sigma}$ are Pauli matrices and $j_l(\rho)$ are spherical Bessel functions. R is the bag radius, and m_q is the quark mass. The energy eigenvalues

$$\omega_{njl} = \sqrt{\frac{x_{njl}^2}{R^2} + m_q^2} \ , \tag{A10}$$

are determined by the roots x_{njl} of the equation

$$j_l(x) = (\bar{l} - l)\sqrt{\frac{\omega_{njl} - m_q}{\omega_{njl} + m_q}}\, j_{\bar{l}}(x) = (\bar{l} - l)\frac{W_-}{W_+}\, j_{\bar{l}}(x) \ . \tag{A11}$$

Instead of $\{j, l\}$ we can use the quantum number $\{\kappa\}$ such that

$$j = |\kappa| - \frac{1}{2} \ , \tag{A12}$$

and

$$l = |\kappa| + \frac{\mathrm{sign}(\kappa) - 1}{2} \ , \tag{A13}$$

$$\bar{l} = |\kappa| - \frac{\mathrm{sign}(\kappa) + 1}{2} \ . \tag{A14}$$

In this case, the wave functions are specified by the quantum numbers $K = \{n, \kappa, j_3\}$ and are of the form

$$q_{n\kappa j_3}(\boldsymbol{r}) = N_\kappa(x_{n\kappa}) \begin{pmatrix} iW_+(n\kappa)\, j_l\left(x_{n\kappa}\frac{r}{R}\right)\ \phi_\kappa^{j_3}(\hat{\boldsymbol{r}}) \\ \\ -\mathrm{sign}(\kappa) W_-(n\kappa)\, j_{\bar{l}}\left(x_{n\kappa}\frac{r}{R}\right)\ \phi_{-\kappa}^{j_3}(\hat{\boldsymbol{r}}) \end{pmatrix} \ , \tag{A15}$$

$$q^c_{n\kappa j_3}(\boldsymbol{r}) = N_\kappa(x_{n\kappa}) \begin{pmatrix} iW_-(n\kappa)\, j_{\bar{l}}\left(x_{n\kappa}\frac{r}{R}\right)\ \phi_{-\kappa}^{j_3}(\hat{\boldsymbol{r}}) \\ \\ -\mathrm{sign}(\kappa) W_+(n\kappa)\, j_l\left(x_{n\kappa}\frac{r}{R}\right)\ \phi_\kappa^{j_3}(\hat{\boldsymbol{r}}) \end{pmatrix} \ , \tag{A16}$$

where the normalization constant is

$$N_\kappa^{-2}(x_{n\kappa}) = \frac{R^3 j_l^2(x_{n\kappa})}{\omega_{n\kappa}(\omega_{n\kappa} - m_q)} \left\{ 2\omega_{n\kappa}\left[\omega_{n\kappa} + \frac{\kappa}{R}\right] + \frac{m_q}{R} \right\} \ , \tag{A17}$$

the angular parts of the wave functions are

$$\begin{aligned}\phi_\kappa^{j_3}(\hat{\boldsymbol{r}}) &= -\mathrm{sign}(\kappa)\sqrt{\frac{|\kappa|+\mathrm{sign}(\kappa)(\frac{1}{2}-j_3)}{2|\kappa|+\mathrm{sign}(\kappa)}}\,Y_l^{j_3-\frac{1}{2}}(\hat{\boldsymbol{r}})\chi^{\frac{1}{2}} \\ &\quad +\sqrt{\frac{|\kappa|+\mathrm{sign}(\kappa)(\frac{1}{2}+j_3)}{2|\kappa|+\mathrm{sign}(\kappa)}}\,Y_l^{j_3+\frac{1}{2}}(\hat{\boldsymbol{r}})\chi^{-\frac{1}{2}}\ ,\end{aligned} \tag{A18}$$

and $\omega_{n\kappa}$'s are given by the equation

$$j_l(x)+\mathrm{sign}(\kappa)\frac{W_-}{W_+}\,j_{\bar{l}}(x)=0\ . \tag{A19}$$

Our conventions follow those of [85].

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